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NUMERICAL INVESTIGATION OF MHD FLOW AND HEAT TRANSFER WITH A REFERENCE TO COPPER-WATER NANO-FLUID

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ABSTRACT

This article presents the magnetohydrodynamic flow and heat transfer of water-based nanofluid in divergent and convergent channels. Equations overseeing the flow are changed to a set of ordinary differential equations by utilizing suitable similitude changes. Resulting system is settled utilizing a solid numerical methodology called Runge-Kutta-Fehlberg method. Results are contrasted and existing solutions accessible in the writing and a brilliant arrangement is seen.

Three states of nanoparticles, specifically, platelet-, chamber , and block formed particles, are considered to play out the investigation. Impact of arising parameters, for example, channel opening, Reynolds number, attractive parameter, Eckert number, and the nanoparticle volume division are heighted with the assistance of diagrams combined with extensive conversations. The attractive field can be utilized as a controlling parameter to lessen the backflow districts for the divergent channel case. Temperature of the fluid can be controlled with the assistance of solid attractive field. It is additionally seen that platelet-formed particles have higher temperature values when contrasted with chamber and block molded particles.

KEYWORDS

Water-based nanofluids, velocity slip, carbon nanotubes, heat transfer, numerical solution, diverging and converging channels

INTRODUCTION

In 1915, Jeffery1 and Hamel2 figured out a problem for the flow between non-equal dividers. These flows are significant due to numerous applications in ventures, clinical and bio-mechanics, and designing. Since the original works, numerous analysts have attempted to broaden the flows in veering and joining channels considering different impacts like magnetohydrodynamics (MHD) and slip, and heat transfer peculiarities.

Newtonian nature of the fluid has been thought of as in the vast majority of these examinations. Change in point assumes a significant part in these flows as talked about in different investigations. Nanofluids, as the name recommends, are the suspensions of nanoparticles. Because of uses in designing, clinical, and businesses, nanotechnology is acquiring its significance.

Customary fluids are awful guides; to adapt up to this problem, nanoparticles are added to the conventional fluids like water, lamp fuel, and ethylene glycol. The expansion of nanoparticles results in improved warm properties of these fluids.

The equations administering the flow under the impact of attractive field are changed into non-direct system of ordinary differential equations. Because of the implausibility of precise solutions and muddled nature of the model for warm conductivity, numerical solution has been gotten utilizing Runge-Kutta-Fehlberg (RKF) method. Results got are plotted against the parameters included combined with thorough conversations. Correlation of current results with previously existing ones demonstrates the proficiency and legitimacy of solutions.

Flow of nanofluid because of source or sink is considered at the convergence of two unbending plates. 2a is taken as the point between the dividers of the channels (Figure 1). Flow is thought to be outspread and symmetric in nature. Base fluid water is thought of, which is immersed by copper nanoparticles.

One more significant peculiarity is the decrease in backflow for expanding attractive powers. More grounded attractive powers result in sped up flow close to the dividers of the channel that consequently decreases the backflow. This procedure is very helpful to diminish the partition for the divergent channel case. It is essential to add that every one of the states of nanoparticles effectsly affect the velocity of the fluid in divergent channel case.

Since Reynolds number is the proportion of energy powers and the gooey powers, this implies that higher values of Re are because of the more grounded force powers.

Because of this explanation, velocity of the fluid will in general diminish and backflow arises close to the dividers of the channel. From this, one can see that the more grounded energy powers might be liable for the division in divergent channels.

All through this article, our emphasis is on water-based nanofluid, and copper is taken as the nanoparticle. The model utilized for the viable warm conductivity of nanofluids is Hamilton and Crosser's model. This model considers different states of nanoparticles.

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Non-round nature of nanoparticles is considered. A cross attractive field of solidarity B0 is additionally applied. Likewise, it is accepted that there is a warm harmony between the base fluid and nanoparticles involved. Under the aforementioned shows, the velocity recorded is the fate of the structure V = ½ur, 0, 0, where ur is a component of both r and u. The equations administering the flow are written in the structure:

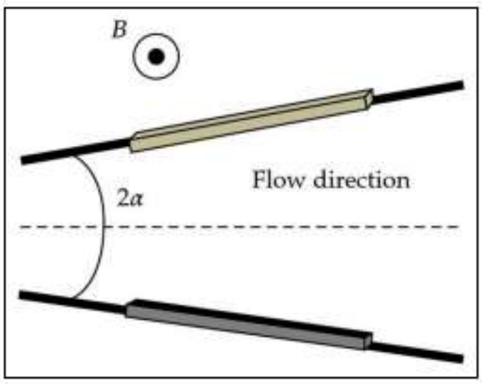

Figure 1. Schematic diagram of the flow problem.

$$\frac{1}{r}\frac{\partial}{\partial r}(r\breve{u}_r) = 0 \tag{1}$$

$$\rho_{nf}\left(\breve{u}_r\frac{\partial \breve{u}_r}{\partial r}\right) = -\frac{\partial p}{\partial r} + \mu_{nf}\left(\frac{\partial^2 \breve{u}_r}{\partial r^2} + \frac{1}{r}\frac{\partial \breve{u}_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \breve{u}_r}{\partial \theta^2} - \frac{\breve{u}_r}{r^2}\right) - \frac{\sigma_{nf}B_0^2(\breve{u}_r)}{r^2} \tag{2}$$

$$-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial \theta} + \frac{2}{r^2}\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial \breve{u}_r}{\partial \theta}\right) = 0 \tag{3}$$

$$\breve{u}_r\frac{\partial \breve{T}}{\partial r} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 \breve{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \breve{T}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \breve{T}}{\partial \theta^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[4\left\{\left(\frac{\partial \breve{u}_r}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial \breve{u}_r}{\partial \theta}\right)^2\right\}\right] \tag{4}$$

Boundary conditions for the problem are as follows

$$\begin{aligned} &\breve{u}_r = U, \quad \frac{\partial \breve{u}_r}{\partial \theta} = 0, \quad \frac{\partial \breve{T}}{\partial \theta} = 0, \quad \text{at}\,\theta = 0 \\ &\breve{u}_r = 0, \quad \breve{T} = \breve{T}_w, \quad \text{at}\,\theta = 0 \end{aligned} \tag{5}$$

In the above equations, ur is the component of the velocity, and T ^ is the temperature of the base fluid.109

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{6}$$

$$\begin{aligned} &(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\ &\frac{\sigma_{nf}}{\sigma_f} = \frac{(\sigma_s + 2\sigma_f) + 2(\sigma_s - \sigma_f)\phi}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\phi} \end{aligned} \tag{7}$$

Here, mf is the viscosity of the base fluid, f is the nanoparticle volume fraction, kf is the thermal conductivity, sf is the electrical conductivity, and rf is the density of the nanoparticles. The model for the effective thermal conductivity of the nanofluid considered here is Hamilton and Crosser's model. When the thermal conductivity of the nanoparticles is 100 times as compared to the base fluid, the effective thermal conductivity of the nanofluid is expressed as:

$$k_{nf} = k_f \left(\frac{k_s + (m+1)k_f - (m+1)(k_f - k_s)\phi}{k_s + (m+1)k_f + (k_f - k_s)\phi} \right) \quad (8)$$

Here, kf and ks represent the conductivities of the base fluid and the nanoparticles, respectively. In Hamilton and Crosser's model, m is the shape factor of the nanoparticles given by 3=c (which is the sphericity defined by the ratio of the surface area of the sphere and the surface area of the real particle with equal volumes). For m = 3, Hamilton and Crosser's model reduces to Maxwell's model. Different shapes of nanoparticles along with the sphericities are presented in Table 1.

Equation (1) can also be written as

$$f(\theta) = r\bar{u}_r(r, \theta) \quad (9)$$

Employing the similarity transform, the above equations can be reduced to the following dimensionless form

$$F(\xi) = \frac{f(\theta)}{f_{max}}, \quad \xi = \frac{\theta}{\alpha}, \quad \beta = \frac{\breve{T}}{\breve{T}_w} \quad (10)$$

Eliminating pressure p from equations (2) and (3) and using equations (9) and (10), the non-linear ordinary differential equations representing velocity and temperature profile will reduce to

$$F''' - 2\alpha Re(1-\phi)^{2.5}\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)FF' + \left(4 - Ha(1-\phi)^{2.5}\frac{\sigma_{nf}}{\sigma_f}\right)\alpha^2 F' = 0 \quad (11)$$

$$\frac{k_{nf}}{k_f}\beta''(\eta) + \frac{EcPr}{(1-\phi)^{2.5}}\left((1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\left[4\alpha^2F^2 + (F')^2\right] = 0 \quad (12)$$

Here, prime denotes the differentiation w.r.t. j. Implementation of similarity transforms reduces the boundary conditions to:

Table 1: Sphericity and shape factor of different nanoparticles

Nanoparticle shapes	Aspect ratio	Sphericity	Shape factor
Platelet	1:1/18	0.52	5.7
Cylinder	1:8	0.62	4.9
Brick	1:1:1	0.81	3.7

$$F(0) = 1, \quad F'(0) = 0, \quad F(1) = 0$$
$$\beta'(0) = 0, \quad \beta(1) = 1 \quad (13)$$

Here, Re is the Reynolds number, which is defined as

$$Re = \frac{f}{\upsilon} = \frac{Ur\alpha}{\upsilon}\begin{pmatrix}\text{Divergent channel}: \alpha>0, U>0 \\ \text{Convergent channel}: \alpha<0, U<0\end{pmatrix} \quad (14)$$

Equations (11) and (12) with the comparing boundary conditions (13) are as two-point boundary value problem. RKF strategy is utilized to tackle the system. Shooting method is first utilized to change over the non-direct system into a set of introductory value problems. Solution is then acquired with the assistance of RKF method. For asymptotically convergent results, a tolerance level of 10^{-6} is restricted.

RESULTS AND DISCUSSIONS

This segment will zero in on the graphical portrayal of the got results. Varieties in velocity and temperature profiles for differing parameters will be plotted.

Table 1 gives the numerical values of various thermophysical properties of the base fluid and nanoparticles. The sphericity of various non-circular nanoparticles is given in Table 2. It is appropriate to make reference to here that the states of nanoparticles in this article are taken as platelet, chamber, and block. Since water is taken as the base fluid, the value of Prandtl number Pr is fixed at 6.2 all through this review. To feature the impacts of involved parameters on velocity and temperature profiles, this segment is partitioned into two sub-segments : One for the divergent channel and other for the convergent channel.

Table 2: Thermophysical properties of base fluid and nanoparticles

	$\rho(kg/m^3)$	$C_p(J/kg\,K)$	$k(W/mk)$	Electrical conductivity $\sigma(\Omega m)_l$
Pure water	997.1	4179	0.613	0.05
Copper (Cu)	8933	385	401	5.96×10^7

Influence of parameters involved on the velocity and temperature profile for divergent channel will be discussed in this section. The variations in velocity for increasing values of channel opening a on velocity profile are depicted in Figure 2.

The expanding values of opening point result in lower values for the velocity as we move close to the dividers of the channel. At the focal piece of the channel, the adjustment of velocity is practically immaterial. One can likewise see that expanding point will result in backflow that consequently may cause partition close to the dividers of the channel. It is relevant to make reference to here that every one of the states of nanoparticles have indistinguishable impact on the velocity of the fluid.

Figure 3 features the deviations in velocity profile for the addition in Reynolds number Re. Close to the dividers of the channel, higher the values of Re, lower is the velocity of the fluid.

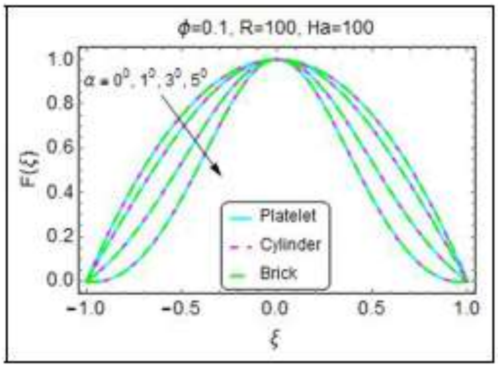

Figure 2. Velocity profile for variation in α.

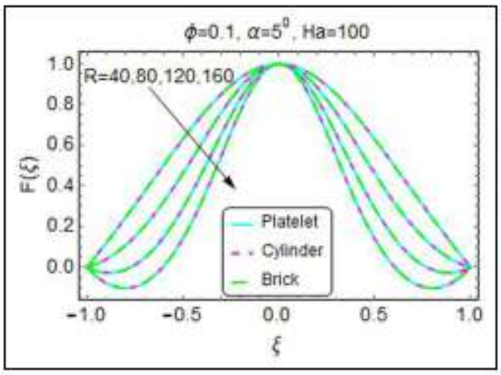

Figure 3. Velocity profile for variation in *Re*.

Changes in velocity with the variety in solid volume part of nanoparticles are plotted in Figure 4. It tends to be seen that higher how much nanoparticles in the fluid, lower will be the velocity for the divergent channel.

This change is unmistakable close to the dividers of the channel and turns out to be more slow at the focal part. The manner by which velocity is impacted by rising values of attractive number Ha is plotted in Figure 5. More grounded attractive powers result in increase for the velocity profile of the fluid.

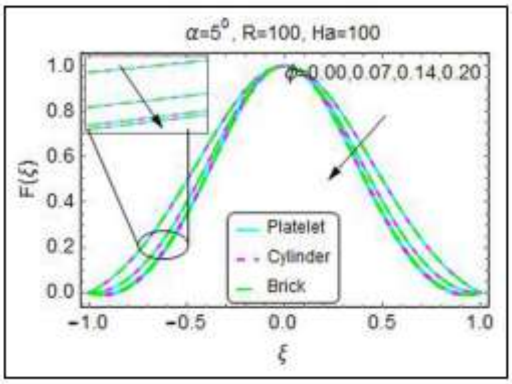

Figure 4. Velocity profile for variation in φ.

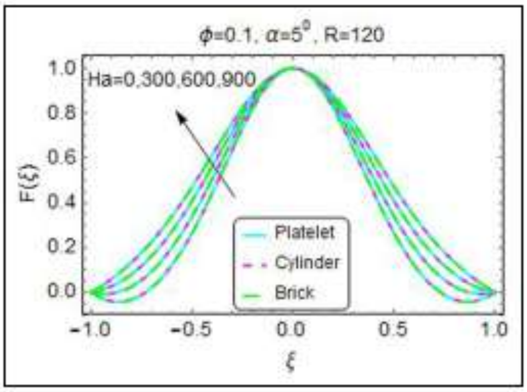

Figure 5. Velocity profile for variation in γ.

CONCLUSION

In this study, Jeffery and Hamel's flow problem is planned for nanofluids with heat transfer impacts. The model utilized for viable warm conductivity of the nanofluids is introduced by considering Hamilton and Crosser's model. Three states of nanoparticles are thought of, specifically, platelet-, chamber , and block molded particles. Water is utilized as base fluid, while copper is taken as nanoparticle. Acquired results are introduced graphically combined with exhaustive conversations. Near investigation verifies the results acquired scientifically and numerically also. The significant results of this study are as per the following:

- Increase in channel opening and the Reynolds number results in backflow for separating channel case;

- This backflow can be decreased by utilizing a solid attractive field.
- Temperature of the fluid can likewise be controlled with the assistance of solid attractive field;
- Temperature of the fluid becomes lower with higher shape factor.

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